

Integration Quiz Review 2

Answer Key:

$$\textcircled{1} \int (3x^2 + 5x - 2) dx$$

$$\boxed{x^3 + \frac{5}{2}x^2 - 2x + C}$$

$$\textcircled{3} \int (3xe^{2x-5} - 4) dx$$

$$3 \int x e^{2x-5} dx - \int 4 dx$$

$$u = 2x - 5 \quad du = 2 dx$$

$$dx = \frac{du}{2}$$

$$3 \int x e^u \frac{du}{2} - 4x + C$$

$$\frac{3}{2} \int e^u du - 4x + C$$

$$\frac{3}{2} e^u - 4x + C$$

$$\boxed{\frac{3}{2} e^{2x-5} - 4x + C}$$

$$\textcircled{2} \int \frac{2x \sqrt[3]{8x} + 5x^2 - 6}{2\sqrt{x}} dx$$

$$\int \frac{4x^{\frac{4}{3}} + 5x^2 - 6}{2x^{\frac{1}{2}}} dx$$

$$\int \left(2x^{\frac{5}{6}} + \frac{5}{2}x^{\frac{3}{2}} - 3x^{-\frac{1}{2}} \right) dx$$

$$\boxed{\frac{12}{11}x^{\frac{11}{6}} + x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + C}$$

$$\textcircled{4} \int (3^{\sin(2x)} \cos(2x) + e^{2x}) dx$$

$$\int 3^{\sin(2x)} \cos(2x) dx + \int e^{2x} dx$$

$$u = \sin(2x)$$

$$du = 2 \cos(2x) dx$$

$$dx = \frac{du}{2 \cos(2x)}$$

$$u = 2x$$

$$du = 2 dx$$

$$dx = \frac{du}{2}$$

$$\int 3^u \cos(2x) \frac{du}{2 \cos(2x)} + \int e^u \frac{du}{2}$$

$$\frac{1}{2} \int 3^u du + \frac{1}{2} \int e^u du$$

$$\frac{1}{2} \frac{3^u}{\ln(3)} + \frac{1}{2} e^u + C$$

$$\boxed{\frac{3^{\sin(2x)}}{2 \ln(3)} + \frac{e^{2x}}{2} + C}$$

$$\textcircled{5} \int \frac{9x + 6}{3x^2 + 4x} dx$$

$$u = 3x^2 + 4x \quad du = (6x + 4) dx$$

$$dx = \frac{du}{6x + 4}$$

$$\int \frac{9x + 6}{u} \frac{du}{6x + 4}$$

$$\int \frac{3(3x+2)}{u} \frac{du}{2(3x+2)}$$

$$\frac{3}{2} \int \frac{1}{u} du = \frac{3}{2} \ln |u| + C$$

$$\boxed{\frac{3}{2} \ln |3x^2 + 4x| + C}$$

$$\textcircled{7} \int \frac{-3 \sec(2x) \sin(2x)}{\cos(2x)} dx$$

$$-3 \int \sec(2x) \tan(2x) dx$$

$$-\frac{3}{2} \int 2 \sec(2x) \tan(2x) dx$$

$$\boxed{-\frac{3}{2} \sec(2x) + C}$$

$$\textcircled{6} \int \frac{3x^2 + 5x - 2}{x + 3} dx$$

$$\begin{array}{r|rr} -3 & 3 & 5 & -2 \\ & 3 & -4 & 10 \end{array}$$

$$\int \left(3x - 4 + \frac{10}{x+3} \right) dx$$

$$\boxed{\frac{3}{2}x^2 - 4x + 10 \ln |x+3| + C}$$

$$\textcircled{8} \int \frac{5x}{\tan(3x^2) \cos(3x^2)} dx$$

$$5 \int \frac{x}{\frac{\sin(3x^2)}{\cos(3x^2)} \cdot \cos(3x^2)} dx$$

$$5 \int x \csc(3x^2) dx \quad \begin{array}{l} u = 3x^2 \\ du = 6x dx \end{array}$$

$$5 \int x \csc(u) \frac{du}{6x} \quad dx = \frac{du}{6x}$$

$$\frac{5}{6} \int \csc u du$$

$$-\frac{5}{6} \ln |\csc u + \cot u| + C$$

$$\boxed{-\frac{5}{6} \ln |\csc(3x^2) + \cot(3x^2)| + C}$$

$$\textcircled{9} \int \csc^3(x) \cot(x) dx$$

$$\int \csc^2(x) \csc(x) \cot(x) dx$$

$$u = \csc(x) \quad du = -\csc(x) \cot(x) dx$$

$$dx = \frac{du}{-\csc(x) \cot(x)}$$

$$\int u^2 \frac{\cancel{\csc(x) \cot(x)}}{\cancel{-\csc(x) \cot(x)}} du$$

$$-\int u^2 du = -\frac{1}{3} u^3 + C$$

$$\boxed{-\frac{1}{3} \csc^3(x) + C}$$

$$\textcircled{10} \int -7x \cot(2x^2) dx$$

$$u = 2x^2 \quad du = 4x dx$$

$$dx = \frac{du}{4x}$$

$$-7 \int x \cot(u) \frac{du}{4x}$$

$$-\frac{7}{4} \int \cot(u) du$$

$$-\frac{7}{4} \ln|\sin(u)| + C$$

$$\boxed{-\frac{7}{4} \ln|\sin(2x^2)| + C}$$

$$\textcircled{11} \int \frac{4}{8 + 4x^2} dx$$

$$\left(\frac{1}{2}\right)^4 \int \frac{1(2)}{(\sqrt{8})^2 + (2x)^2} dx$$

$$2 \int \frac{2}{(\sqrt{8})^2 + (2x)^2} dx$$

$$\frac{2}{\sqrt{8}} \arctan\left(\frac{2x}{\sqrt{8}}\right) + C$$

$$\boxed{\frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) + C}$$

or

$$\boxed{\frac{\sqrt{2}}{2} \arctan\left(\frac{x\sqrt{2}}{2}\right) + C}$$

$$\textcircled{12} \int \frac{2x}{\sqrt{9 - 5x^4}} dx$$

$$\left(\frac{1}{2\sqrt{5}}\right) 2 \int \frac{x(2\sqrt{5})}{\sqrt{3^2 - (x^2\sqrt{5})^2}} dx$$

$$\boxed{\frac{1}{\sqrt{5}} \arcsin\left(\frac{x^2\sqrt{5}}{3}\right) + C}$$

$$(13) \int 3x^2 \sqrt{4x-1} \, dx$$

$$u = 4x - 1$$

$$dx = \frac{du}{4}$$

$$du = 4 \, dx$$

$$x = \frac{u+1}{4}$$

$$\int 3 \left(\frac{u+1}{4} \right)^2 u^{\frac{1}{2}} \frac{du}{4}$$

$$\frac{3}{4} \int \left(\frac{u^2 + 2u + 1}{16} \right) u^{\frac{1}{2}} du = \frac{3}{64} \int \left(u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}} \right) du$$

$$\frac{3}{64} \left[\frac{2}{7} u^{\frac{7}{2}} + \frac{4}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} \right] + C$$

$$\frac{6}{6720} u^{\frac{1}{2}} (15u^3 + 42u^2 + 35u) + C$$

$$\boxed{\frac{1}{1120} \sqrt{4x-1} (15(4x-1)^3 + 42(4x-1)^2 + 35(4x-1)) + C}$$

$$(14) \int \frac{3x}{\sqrt[3]{5x+3}} \, dx$$

$$u = 5x + 3$$

$$dx = \frac{du}{5}$$

$$du = 5 \, dx$$

$$x = \frac{u-3}{5}$$

$$3 \int \left(\frac{u-3}{5} \right) u^{-\frac{1}{3}} \frac{du}{5} = \frac{3}{25} \int \left(u^{\frac{2}{3}} - 3u^{-\frac{1}{3}} \right) du$$

$$\frac{3}{25} \left[\frac{3}{5} u^{\frac{5}{3}} - \frac{9}{2} u^{\frac{2}{3}} \right] + C = \frac{9}{250} u^{\frac{2}{3}} (2u - 15) + C$$

$$\frac{9}{250} (5x+3)^{\frac{2}{3}} [2(5x+3) - 15] + C = \boxed{\frac{9}{250} (5x+3)^{\frac{2}{3}} (10x-9) + C}$$